

Cosmological Evolution of Hessece Dark Energy and Avoidance of the Big Rip

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ABSTRACT

Recently, many dark energy models whose equation-of-state parameter can cross the phantom divide $w_{de} = -1$ have been proposed. In a previous paper [Class. Quant. Grav. **22**, 3189 (2005); hep-th/0501160], we suggest such a model named hessence, in which a non-canonical complex scalar field plays the role of dark energy. In this work, the cosmological evolution of the hessence dark energy is investigated. We consider two cases: one is the hessence field with an exponential potential, and the other is with a (inverse) power law potential. We separately investigate the dynamical system with four different interaction forms between hessence and background perfect fluid. It is found that the big rip never appears in the hessence model, even in the most general case, beyond particular potentials and interaction forms.

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I. INTRODUCTION

Dark energy [1] has been one of the focuses in modern cosmology since the discovery of accelerated expansion of the universe [2, 3, 4, 5, 6, 40]. The simplest candidate of dark energy is a tiny positive cosmological constant. As an alternative to the cosmological constant, some dynamical scalar field models have been proposed, such as quintessence [7, 43], phantom [8, 9, 10, 41], k-essence [11] etc. In the observational cosmology of dark energy, equation-of-state parameter (EoS) $w_{de} \equiv p_{de}/\rho_{de}$ plays a central role, where p_{de} and ρ_{de} are the pressure and energy density of dark energy respectively. The most important difference between cosmological constant and dynamical scalar fields is that the EoS of the former is always a constant, -1 , while the EoS of the latter can be variable during the evolution of the universe.

Recently, by fitting the SNe Ia data, marginal evidence for $w_{de}(z) < -1$ at $z < 0.2$ has been found [12]. In addition, many best-fits of the present value of w_{de} are less than -1 in various data fittings with different parameterizations (see [13] for a recent review). The present data seem to slightly favor an evolving dark energy with w_{de} being below -1 around present epoch from $w_{de} > -1$ in the near past [14]. Obviously, the EoS w_{de} cannot cross the so-called phantom divide $w_{de} = -1$ for quintessence or phantom alone. Some efforts [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] have been made to build dark energy model whose EoS can cross the phantom divide. Here we mention some of them, such as the geometric approach (brane or string) [18], holographic dark energy [19, 20], and scalar-tensor theory (esp. non-minimal coupled scalar field to gravity) [27, 42], etc.

Although some variants of k-essence [11] look possible to give a promising solution to cross the phantom divide, a no-go theorem, shown in [21], shatters this kind of hopes: it is impossible to cross the phantom divide $w_{de} = -1$, provided that the following conditions are satisfied: (i) classical level, (ii) general relativity is valid, (iii) single real scalar field, (iv) arbitrary Lagrangian density $p(\varphi, X)$, where $X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ is the kinetic energy term, (v) $p(\varphi, X)$ is continuous function and is differentiable enough, and (vi) the scalar field is minimal coupled to gravity. Thus, to implement the transition from $w_{de} > -1$ to $w_{de} < -1$ or vice versa, it is necessary to give up at least one of conditions mentioned above.

Obviously, the simplest way to get around this no-go theorem is to consider a model with two real scalar fields, i.e. to break the third condition. Feng, Wang and Zhang in [14] proposed a so-called quintom model which is a hybrid of quintessence and phantom (so the name quintom). Naively, one may consider a Lagrangian density [14, 22, 23, 24]

$$\mathcal{L}_{quintom} = \frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}(\partial_\mu\phi_2)^2 - V(\phi_1, \phi_2), \quad (1)$$

where ϕ_1 and ϕ_2 are two real scalar fields and play the roles of quintessence and phantom respectively. Considering a spatially flat Friedmann-Robertson-Walker (FRW) universe and assuming the scalar fields ϕ_1 and ϕ_2 are homogeneous, one obtains the *effective* pressure and energy density for the quintom

$$p_{quintom} = \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 - V(\phi_1, \phi_2), \quad \rho_{quintom} = \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 + V(\phi_1, \phi_2), \quad (2)$$

respectively. The corresponding *effective* EoS is given by

$$w_{quintom} = \frac{\dot{\phi}_1^2 - \dot{\phi}_2^2 - 2V(\phi_1, \phi_2)}{\dot{\phi}_1^2 - \dot{\phi}_2^2 + 2V(\phi_1, \phi_2)}. \quad (3)$$

It is easy to see that $w_{quintom} \geq -1$ when $\dot{\phi}_1^2 \geq \dot{\phi}_2^2$ while $w_{quintom} < -1$ when $\dot{\phi}_1^2 < \dot{\phi}_2^2$. The cosmological evolutions of the quintom model without direct coupling between ϕ_1 and ϕ_2 , i.e. $V(\phi_1, \phi_2) = V_{\phi_1} + V_{\phi_2} \equiv V_{\phi_1 0} \exp(-\lambda_{\phi_1} \kappa \phi_1) + V_{\phi_2 0} \exp(-\lambda_{\phi_2} \kappa \phi_2)$, and with a special interaction between ϕ_1 and ϕ_2 , i.e. $V(\phi_1, \phi_2) = V_{\phi_1} + V_{\phi_2} + V_{int}$ and $V_{int} \sim (V_{\phi_1} V_{\phi_2})^{1/2}$, were studied by Guo *et al* [22] and Zhang *et al* [23], respectively. They showed that the transition from $w_{quintom} > -1$ to $w_{quintom} < -1$ or vice versa is possible in this type of quintom model.

In [28], by a new view of quintom dark energy, we proposed a novel non-canonical complex scalar field, which was named “hessence”, to play the role of quintom. In the hessence model, the phantom-like role is played by the so-called internal motion θ , where θ is the internal degree of freedom of hessence. The transition from $w_h > -1$ to $w_h < -1$ or vice versa is also possible in the hessence model [28]. We will briefly present the main points of hessence model in Sec. II.

The main aim of this work is to investigate the cosmological evolution of hessence dark energy. We consider the hessence and background perfect fluid as a dynamical system [29]. By the phase-space analysis, we find that some stable attractors can exist, which are either scaling solutions or hessence-dominated solutions with EoS (w_h and w_{eff}) larger than or equal to -1 . No phantom-like late time attractors with EoS less than -1 exist. This result is very different from the quintom model considered in [22, 23], where the phantom-dominated solution is the *unique* late time attractor. If the universe is attracted into the unique phantom-dominated attractor with EoS less than -1 , it will undergo a super-accelerated expansion (i.e. $\dot{H} > 0$) forever and the big rip is inevitable [9] (see also [16, 30, 39]). On the contrary, in the hessence model, EoS less than -1 is transient. Eventually, it will come back to quintessence-like attractors whose EoS is larger than -1 or asymptotically to de Sitter attractor whose EoS is a constant -1 . Therefore, the big rip will not appear in the hessence model.

The plan of this paper is as follows. In Sec. II, we will briefly present the main points of the hessence model. In Sec. III, we give out the equations of the dynamical system of hessence with/without interaction to background perfect fluid for the most general case. That is, we leave the potential of hessence and the interaction form undetermined. We will investigate the dynamical system for the models with exponential and (inverse) power law potentials in Sec. IV and Sec. V, respectively. In each case with different potential, we consider four different interaction forms between hessence and background perfect fluid. The first one corresponds to the case without interaction, i.e. the interaction term is zero. The other three forms are taken to be the most familiar interaction ones considered in the literature. In all these cases, we find that *no* phantom-like late time attractors with EoS less than -1 exist. In Sec. VI, we will show the big rip will not appear in a general case beyond the above considered cases. Finally, brief conclusion and discussions are given in Sec. VII.

We use the units $\hbar = c = 1$, $\kappa^2 \equiv 8\pi G$ and adopt the metric convention as $(+, -, -, -)$ throughout this paper.

II. HESSENCE DARK ENERGY

Following [28], we consider a non-canonical complex scalar field as the dark energy, namely hessence,

$$\Phi = \phi_1 + i\phi_2, \quad (4)$$

with a Lagrangian density

$$\mathcal{L}_h = \frac{1}{4} [(\partial_\mu \Phi)^2 + (\partial_\mu \Phi^*)^2] - U(\Phi^2 + \Phi^{*2}) = \frac{1}{2} [(\partial_\mu \phi)^2 - \phi^2(\partial_\mu \theta)^2] - V(\phi), \quad (5)$$

where we have introduced two new variables (ϕ, θ) to describe the hessence, i.e.

$$\phi_1 = \phi \cosh \theta, \quad \phi_2 = \phi \sinh \theta, \quad (6)$$

which are defined by

$$\phi^2 = \phi_1^2 - \phi_2^2, \quad \coth \theta = \frac{\phi_1}{\phi_2}. \quad (7)$$

Considering a spatially flat FRW universe with scale factor $a(t)$ and assuming ϕ and θ are homogeneous, from Eq. (5) we obtain the equations of motion for ϕ and θ ,

$$\ddot{\phi} + 3H\dot{\phi} + \phi\dot{\theta}^2 + V_{,\phi} = 0, \quad (8)$$

$$\phi^2\ddot{\theta} + (2\phi\dot{\phi} + 3H\phi^2)\dot{\theta} = 0, \quad (9)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, a dot and the subscript “ $_{,\phi}$ ” denote the derivatives with respect to cosmic time t and ϕ , respectively. The pressure and energy density of the hessence are

$$p_h = \frac{1}{2} (\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi), \quad \rho_h = \frac{1}{2} (\dot{\phi}^2 - \phi^2\dot{\theta}^2) + V(\phi), \quad (10)$$

respectively. Eq. (9) implies

$$Q = a^3\phi^2\dot{\theta} = \text{const.} \quad (11)$$

which is associated with the total conserved charge within the physical volume due to the internal symmetry [28]. It turns out

$$\dot{\theta} = \frac{Q}{a^3 \phi^2}. \quad (12)$$

Substituting this into Eqs. (8) and (10), we can recast them as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{Q^2}{a^6 \phi^3} + V_{,\phi} = 0, \quad (13)$$

$$p_h = \frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6 \phi^2} - V(\phi), \quad \rho_h = \frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6 \phi^2} + V(\phi). \quad (14)$$

It is worth noting that Eq. (13) is equivalent to the energy conservation equation of hessence $\dot{\rho}_h + 3H(\rho_h + p_h) = 0$. The Friedmann equation and Raychaudhuri equation are given by, respectively,

$$H^2 = \frac{\kappa^2}{3}(\rho_h + \rho_m), \quad (15)$$

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_h + \rho_m + p_h + p_m), \quad (16)$$

where p_m and ρ_m are the pressure and energy density of background matter, respectively. The EoS of hessence $w_h \equiv p_h/\rho_h$. It is easy to see that $w_h \geq -1$ when $\dot{\phi}^2 \geq Q^2/(a^6 \phi^2)$ while $w_h < -1$ when $\dot{\phi}^2 < Q^2/(a^6 \phi^2)$. The transition occurs when $\dot{\phi}^2 = Q^2/(a^6 \phi^2)$.

There are other interesting points in the hessence model, such as the avoidance of Q-ball formation, the novel possibility of anti-dark energy, the possible relation between hessence and Chaplygin gas, etc. We refer to the original paper [28] for more details.

III. DYNAMICAL SYSTEM OF HESSENCE WITH/WITHOUT INTERACTION TO BACKGROUND PERFECT FLUID

Now, we generalize the original hessence model [28] to a more extensive case. We consider a universe containing both hessence dark energy and background matter. The background matter is described by a perfect fluid with barotropic equation of state

$$p_m = w_m \rho_m \equiv (\gamma - 1)\rho_m, \quad (17)$$

where the so-called barotropic index γ is a constant and satisfies $0 < \gamma \leq 2$. In particular, $\gamma = 1$ and $4/3$ correspond to dust matter and radiation, respectively.

We assume the hessence and background matter interact through an interaction term C , according to

$$\dot{\rho}_h + 3H(\rho_h + p_h) = -C, \quad (18)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = C, \quad (19)$$

which preserves the total energy conservation equation $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$. Clearly, $C = 0$ corresponds to *no* interaction between hessence and background matter. It is worth noting that Eq. (13) should be changed when $C \neq 0$, a new term due to C will appear in the right hand side. Since θ is the internal degree of freedom [28], Eqs. (9), (11) and (12) still hold. The hessence interacts to external matter only through ϕ . Thus, the equation of motion of ϕ should be changed by the interaction.

Following [31, 32, 33], we introduce following dimensionless variables

$$x \equiv \frac{\kappa \dot{\phi}}{\sqrt{6}H}, \quad y \equiv \frac{\kappa \sqrt{V}}{\sqrt{3}H}, \quad z \equiv \frac{\kappa \sqrt{\rho_m}}{\sqrt{3}H}, \quad u \equiv \frac{\sqrt{6}}{\kappa \phi}, \quad v \equiv \frac{\kappa}{\sqrt{6}H} \frac{Q}{a^3 \phi}. \quad (20)$$

By the help of Eqs. (14)–(16), the evolution equations (18) and (19) can then be rewritten as a dynamical system:

$$x' = 3x \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - 1 \right) - uv^2 - \frac{\kappa V_{,\phi}}{\sqrt{6}H^2} - C_1, \quad (21)$$

$$y' = 3y \left(x^2 - v^2 + \frac{\gamma}{2} z^2 \right) + \frac{\kappa}{2\sqrt{3}H} \frac{V_{,\phi}}{\sqrt{V}} \frac{\dot{\phi}}{H}, \quad (22)$$

$$z' = 3z \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - \frac{\gamma}{2} \right) + C_2, \quad (23)$$

$$u' = -xu^2, \quad (24)$$

$$v' = 3v \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - 1 \right) - xuv, \quad (25)$$

where

$$C_1 \equiv \frac{\kappa C}{\sqrt{6}H^2\dot{\phi}}, \quad C_2 \equiv \frac{\kappa C}{2\sqrt{3}H^2\sqrt{\rho_m}}, \quad (26)$$

a prime denotes derivative with respect to the so-called e-folding time $\mathcal{N} \equiv \ln a$, and we have used

$$-\frac{\dot{H}}{H^2} = 3 \left(x^2 - v^2 + \frac{\gamma}{2} z^2 \right). \quad (27)$$

The Friedmann constraint equation (15) becomes

$$x^2 + y^2 + z^2 - v^2 = 1. \quad (28)$$

The fractional energy densities of hessence and background matter are given by

$$\Omega_h = x^2 + y^2 - v^2, \quad \Omega_m = z^2, \quad (29)$$

respectively. The EoS of hessence and the effective EoS of the whole system are

$$w_h = \frac{p_h}{\rho_h} = \frac{x^2 - v^2 - y^2}{x^2 - v^2 + y^2}, \quad w_{eff} = \frac{p_h + p_m}{\rho_h + \rho_m} = x^2 - v^2 - y^2 + (\gamma - 1)z^2, \quad (30)$$

respectively. One can see that $w_h \geq -1$ as $x^2 \geq v^2$, while $w_h < -1$ as $x^2 < v^2$. Finally, it is worth noting that $y \geq 0$ and $z \geq 0$ by definition, and in what follows, we only consider the case of expanding universe with $H > 0$.

It is easy to see that Eqs. (21)–(25) become an autonomous system when the potential $V(\phi)$ is chosen to be an exponential or (inverse) power law potential and the interaction term C is chosen to be a suitable form. Indeed, we will consider the model with an exponential or (inverse) power law potential in Sec. IV and Sec. V, respectively. In each model with different potential, we consider four cases with different interaction forms between hessence and background perfect fluid. The first case is the one without interaction, i.e. $C = 0$. The other three cases are taken as the most familiar interaction terms extensively considered in the literature:

$$\text{Case (I)} \quad C = 0,$$

$$\text{Case (II)} \quad C = \alpha\kappa\rho_m\dot{\phi},$$

$$\text{Case (III)} \quad C = 3\beta H\rho_{tot} = 3\beta H(\rho_h + \rho_m),$$

$$\text{Case (IV)} \quad C = 3\eta H\rho_m,$$

where α , β and η are dimensionless constants. The interaction form Case (II) arises from, for instance, string theory or scalar-tensor theory (including Brans-Dicke theory) [33, 34, 35]. The interaction forms Case (III) [36] and Case (IV) [37] are phenomenally proposed to alleviate the coincidence problem.

In the next two sections, we first obtain the critical points $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ of the autonomous system by imposing the conditions $\bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = \bar{v}' = 0$. Of course, they are subject to the Friedmann constraint, i.e. $\bar{x}^2 + \bar{y}^2 + \bar{z}^2 - \bar{v}^2 = 1$. We then discuss the existence and stability of these critical points. An attractor is one of the stable critical points of the autonomous system.

IV. MODEL WITH EXPONENTIAL POTENTIAL

In this section, we consider the hessence model with an exponential potential

$$V(\phi) = V_0 e^{-\lambda\kappa\phi}, \quad (31)$$

where λ is a dimensionless constant. Without loss of generality, we choose λ to be positive, since we can make it positive through field redefinition $\phi \rightarrow -\phi$ if λ is negative. In this case, Eqs. (21)–(25) become

$$x' = 3x \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - 1 \right) - uv^2 + \sqrt{\frac{3}{2}} \lambda y^2 - C_1, \quad (32)$$

$$y' = 3y \left(x^2 - v^2 + \frac{\gamma}{2} z^2 \right) - \sqrt{\frac{3}{2}} \lambda xy, \quad (33)$$

$$z' = 3z \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - \frac{\gamma}{2} \right) + C_2, \quad (34)$$

$$u' = -xu^2, \quad (35)$$

$$v' = 3v \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - 1 \right) - xuv, \quad (36)$$

where C_1 and C_2 are defined by Eq. (26) and depend on the interaction form C . In the following subsections, we will consider different interaction forms C mentioned in the end of Sec. III.

To study the stability of the critical points of Eqs. (32)–(36), we substitute linear perturbations $x \rightarrow \bar{x} + \delta x$, $y \rightarrow \bar{y} + \delta y$, $z \rightarrow \bar{z} + \delta z$, $u \rightarrow \bar{u} + \delta u$, and $v \rightarrow \bar{v} + \delta v$ about the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (32)–(36) and linearize them. Note that these critical points must satisfy the Friedmann constraint, $\bar{y} \geq 0$, $\bar{z} \geq 0$ and requirement of \bar{x} , \bar{y} , \bar{z} , \bar{u} , \bar{v} all being real. Because of the Friedmann constraint (28), there are only four independent evolution equations:

$$\begin{aligned} \delta x' = & - \left[3\bar{y}^2 + 3 \left(1 - \frac{\gamma}{2} \right) \bar{z}^2 + 2\bar{u}\bar{x} \right] \delta x + \left(\sqrt{6}\lambda - 6\bar{x} - 2\bar{u} \right) \bar{y}\delta y \\ & + [3(\gamma - 2)\bar{x} - 2\bar{u}] \bar{z}\delta z - (\bar{x}^2 + \bar{y}^2 + \bar{z}^2 - 1) \delta u - \delta C_1, \end{aligned} \quad (37)$$

$$\delta y' = -\sqrt{\frac{3}{2}} \lambda \bar{y} \delta x + 3 \left[1 - 3\bar{y}^2 + \left(\frac{\gamma}{2} - 1 \right) \bar{z}^2 - \frac{\lambda}{\sqrt{6}} \bar{x} \right] \delta y + 3(\gamma - 2) \bar{y} \bar{z} \delta z, \quad (38)$$

$$\delta z' = -6\bar{y} \bar{z} \delta y + 3 \left[\left(1 - \frac{\gamma}{2} \right) - \bar{y}^2 + 3 \left(\frac{\gamma}{2} - 1 \right) \bar{z}^2 \right] \delta z + \delta C_2, \quad (39)$$

$$\delta u' = -\bar{u}^2 \delta x - 2\bar{x} \bar{u} \delta u, \quad (40)$$

where δC_1 and δC_2 are the linear perturbations coming from C_1 and C_2 , respectively. The four eigenvalues of the coefficient matrix of the above equations determine the stability of the critical point.

A. Case (I) $C = 0$

$C = 0$ means *no* interaction between hessence and background matter. In this case, one has $C_1 = C_2 = 0$. It is easy to find out all critical points $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ of the autonomous system (32)–(36). They are required to be real and satisfy the Friedmann constraint and $\bar{y} \geq 0$, $\bar{z} \geq 0$. We present them in Table I. Next we consider the stabilities of these critical points. In this case, $\delta C_1 = \delta C_2 = 0$. Substituting δC_1 , δC_2 and the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (37)–(40), we find that Points (E.I.1), (E.I.2), (E.I.3) and (E.I.4) are always unstable, while Point (E.I.5) and Point (E.I.6) exist and are stable under condition $\lambda > \sqrt{3\gamma}$ and $\lambda < \sqrt{3\gamma}$, respectively.

The late time attractor (E.I.5) has

$$\Omega_h = \frac{3\gamma}{\lambda^2}, \quad \Omega_m = 1 - \frac{3\gamma}{\lambda^2}, \quad w_h = -1 + \gamma, \quad w_{eff} = -1 + \gamma, \quad (41)$$

which is a scaling solution. The late time attractor (E.I.6) has

$$\Omega_h = 1, \quad \Omega_m = 0, \quad w_h = -1 + \frac{\lambda^2}{3}, \quad w_{eff} = -1 + \frac{\lambda^2}{3}, \quad (42)$$

which is a hessence-dominated solution. Note that their EoS are all larger than -1 .

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$
E.I.1	$\bar{x}^2 \geq 1, 0, 0, 0, \pm\sqrt{\bar{x}^2 - 1}$
E.I.2	$0, 0, 1, \text{any}, 0$
E.I.3	$\pm 1, 0, 0, 0, 0$
E.I.4	$\frac{\sqrt{6}}{\lambda}, 0, 0, 0, \pm\sqrt{\frac{6}{\lambda^2} - 1}$
E.I.5	$\sqrt{\frac{3}{2}\frac{\gamma}{\lambda}}, \sqrt{\frac{3\gamma}{\lambda^2} - \frac{3\gamma^2}{2\lambda^2}}, \sqrt{1 - \frac{3\gamma}{\lambda^2}}, 0, 0$
E.I.6	$\frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}}, 0, 0, 0$

TABLE I: Critical points for Case (I) $C = 0$ in the model with exponential potential.

B. Case (II) $C = \alpha\kappa\rho_m\dot{\phi}$

In this case, $C_1 = \sqrt{\frac{3}{2}}\alpha z^2$ and $C_2 = \sqrt{\frac{3}{2}}\alpha xz$. The physically reasonable critical points of the autonomous system (32)–(36) are summarized in Table II. Next let us consider the stability of these critical points. In this case, $\delta C_1 = \sqrt{6}\alpha\bar{z}\delta z$ and $\delta C_2 = \sqrt{\frac{3}{2}}\alpha\bar{z}\delta x + \sqrt{\frac{3}{2}}\alpha\bar{x}\delta z$. Substituting δC_1 , δC_2 and the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (37)–(40), we find that Point (E.II.1) exists and is stable under condition $\alpha < 0$ and $\bar{x} > \max\left\{1, \frac{\sqrt{6}}{\lambda}, \sqrt{\frac{3}{2}\frac{\gamma-2}{\alpha}}\right\}$; Point (E.II.2p) exists and is stable under condition $\alpha < \sqrt{\frac{3}{2}}(\gamma - 2)$ and $\lambda > \sqrt{6}$; Point (E.II.2m) is always unstable; Point (E.II.3) exists and is stable under condition $\lambda > \sqrt{6\gamma(2-\gamma)}$ and $\sqrt{\frac{3}{2}}(\gamma-2) < \alpha < \sqrt{\frac{3}{2}}(2-\gamma)$ and $\frac{-\lambda - \sqrt{\lambda^2 - 6\gamma(2-\gamma)}}{2} < \alpha < \frac{-\lambda + \sqrt{\lambda^2 - 6\gamma(2-\gamma)}}{2}$; Point (E.II.4) exists and is stable under condition $\lambda < \sqrt{6}$ and $\alpha < (\frac{\gamma}{2} - 1)\lambda$; Point (E.II.5) exists and is stable under condition $0 > \alpha > \max\left\{\sqrt{\frac{3}{2}}(\gamma - 2), (\frac{\gamma}{2} - 1)\lambda\right\}$; Point (E.II.6) exists and is stable in a suitable parameter-space [38]; and Point (E.II.7) exists and is stable under condition $\lambda < \sqrt{6}$ and $\alpha < \frac{3\gamma}{\lambda} - \lambda$. The corresponding Ω_h , Ω_m , w_h and w_{eff} of the attractors are presented in Table II as well. Again, we see that their EoS of hessence and effective EoS are always larger than -1 [note that in Point (E.II.6), $\alpha(\lambda + \alpha) + 3\gamma > 0$ and $\alpha + \lambda > 0$ are required by the existence of its corresponding \bar{y} and \bar{z} , respectively]. This implies that the big rip singularity will not appear in this case.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$	Ω_h	Ω_m	w_h	w_{eff}
E.II.1	$\bar{x}^2 \geq 1, 0, 0, 0, \pm\sqrt{\bar{x}^2 - 1}$	1	0	1	1
E.II.2p	$+1, 0, 0, 0, 0$	1	0	1	1
E.II.2m	$-1, 0, 0, 0, 0$	—	—	—	—
E.II.3	$\sqrt{\frac{2}{3}\frac{\alpha}{\gamma-2}}, 0, \sqrt{1 - \frac{2\alpha^2}{3(\gamma-2)^2}}, 0, 0$	$\frac{2\alpha^2}{3(\gamma-2)^2}$	$1 - \frac{2\alpha^2}{3(\gamma-2)^2}$	1	$-1 + \gamma + \frac{2\alpha^2}{3(2-\gamma)}$
E.II.4	$\frac{\sqrt{6}}{\lambda}, 0, 0, 0, \pm\sqrt{\frac{6}{\lambda^2} - 1}$	1	0	1	1
E.II.5	$\sqrt{\frac{3}{2}\frac{\gamma-2}{\alpha}}, 0, 0, 0, \pm\sqrt{\frac{3(\gamma-2)^2}{2\alpha^2} - 1}$	1	0	1	1
E.II.6	$\sqrt{\frac{3}{2}\frac{\gamma}{\lambda+\alpha}}, \sqrt{\frac{2\alpha^2 - 3(\gamma-2)\gamma + 2\alpha\lambda}{2(\alpha+\lambda)^2}}, \sqrt{\frac{\lambda(\lambda+\alpha) - 3\gamma}{(\lambda+\alpha)^2}}, 0, 0$	$\frac{\alpha(\lambda+\alpha) + 3\gamma}{(\lambda+\alpha)^2}$	$\frac{\lambda(\lambda+\alpha) - 3\gamma}{(\lambda+\alpha)^2}$	$-1 + \frac{3\gamma^2}{\alpha(\lambda+\alpha) + 3\gamma}$	$-1 + \frac{\gamma\lambda}{\alpha+\lambda}$
E.II.7	$\frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}}, 0, 0, 0$	1	0	$-1 + \frac{\lambda^2}{3}$	$-1 + \frac{\lambda^2}{3}$

TABLE II: Critical points for Case (II) $C = \alpha\kappa\rho_m\dot{\phi}$ in the model with exponential potential.

C. Case (III) $C = 3\beta H \rho_{tot} = 3\beta H (\rho_h + \rho_m)$

In this case, $C_1 = \frac{3}{2}\beta x^{-1}$ and $C_2 = \frac{3}{2}\beta z^{-1}$. The physically reasonable critical points of the autonomous system (32)–(36) are presented in Table III, where

$$r_3 \equiv \sqrt{1 + \frac{4\beta}{2-\gamma}}. \quad (43)$$

Points (E.III.4), (E.III.5) and (E.III.6) are the three real solutions of

$$\frac{2}{\gamma} \bar{x} \left(\frac{\lambda}{\sqrt{6}} - \bar{x} \right) \left(\gamma - \sqrt{\frac{2}{3}} \lambda \bar{x} \right) = \beta, \quad (44)$$

$$\bar{y}^2 = \left(\frac{2}{\gamma} - 1 \right) \bar{x}^2 - \sqrt{\frac{2}{3}} \frac{\lambda}{\gamma} \bar{x} + 1, \quad (45)$$

$$\bar{z}^2 = \frac{2}{\gamma} \left(-\bar{x}^2 + \frac{\lambda}{\sqrt{6}} \bar{x} \right), \quad (46)$$

with $\bar{u} = \bar{v} = 0$ and requiring $\bar{y} \geq 0$, $\bar{z} \geq 0$ by definition. Although the real roots of the Eqs. (44)–(46) are easy to obtain, we do not present them here, since they are long in length. Note that Point (E.III.3) exists only when $\beta > 0$ and $\gamma > 2$, which is beyond the range $0 < \gamma \leq 2$.

Next, we consider the stabilities of these critical points. In this case, $\delta C_1 = -\frac{3}{2}\beta \bar{x}^{-2} \delta x$ and $\delta C_2 = -\frac{3}{2}\beta \bar{z}^{-2} \delta z$. Substituting δC_1 , δC_2 and the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (37)–(40), we see that Points (E.III.1p), (E.III.1m) and (E.III.2m) are always unstable, while Point (E.III.2p) exists and is stable under condition $0 > \beta > (\gamma - 2)/4$ and $\lambda > [12 + 3(1 + r_3)(-2 + \gamma)] / (2\sqrt{3}\sqrt{1 - r_3})$. Points (E.III.4), (E.III.5) and (E.III.6) exist and are stable in proper parameter-space [38].

The late time attractor (E.III.2p) has

$$\Omega_h = \frac{1}{2}(1 - r_3), \quad \Omega_m = \frac{1}{2}(1 + r_3), \quad w_h = 1, \quad w_{eff} = 1 + \frac{1}{2}(\gamma - 2)(1 + r_3), \quad (47)$$

which is a scaling solution. The late time attractors (E.III.4), (E.III.5) and (E.III.6) have

$$\Omega_h = \bar{x}^2 + \bar{y}^2, \quad \Omega_m = \bar{z}^2, \quad w_h = -1 + \frac{2\bar{x}^2}{\bar{x}^2 + \bar{y}^2}, \quad w_{eff} = -1 + \sqrt{\frac{2}{3}} \lambda \bar{x}. \quad (48)$$

They are all scaling solutions. It is easy to see that their EoS of hessence and effective EoS are all larger than -1 [note that $0 < \gamma \leq 2$ and $0 < r_3 < 1$ for Point (E.III.2p), while $\bar{x} \geq 0$ is required by Eq. (46) for Points (E.III.4), (E.III.5) and (E.III.6)].

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$
E.III.1p	$[\frac{1}{2}(1 + r_3)]^{1/2}, 0, [\frac{1}{2}(1 - r_3)]^{1/2}, 0, 0$
E.III.1m	$-[\frac{1}{2}(1 + r_3)]^{1/2}, 0, [\frac{1}{2}(1 - r_3)]^{1/2}, 0, 0$
E.III.2p	$[\frac{1}{2}(1 - r_3)]^{1/2}, 0, [\frac{1}{2}(1 + r_3)]^{1/2}, 0, 0$
E.III.2m	$-[\frac{1}{2}(1 - r_3)]^{1/2}, 0, [\frac{1}{2}(1 + r_3)]^{1/2}, 0, 0$
E.III.3	$\frac{\sqrt{6}}{\lambda}, \sqrt{\frac{\beta}{2}}, \sqrt{\frac{\beta}{-2+\gamma}}, 0, \pm \sqrt{-1 + \frac{\beta\gamma}{-4+2\gamma} + \frac{6}{\lambda^2}}$
Note:	For Points (E.III.4), (E.III.5) and (E.III.6), see text

TABLE III: Critical points for Case (III) $C = 3\beta H \rho_{tot} = 3\beta H (\rho_h + \rho_m)$ in the model with exponential potential. r_3 is given in Eq. (43).

D. Case (IV) $C = 3\eta H\rho_m$

In this case, $C_1 = \frac{3}{2}\eta x^{-1}z^2$ and $C_2 = \frac{3}{2}\eta z$. We show the physically reasonable critical points of the autonomous system (32)–(36) in Table IV, where

$$r_y \equiv \sqrt{\frac{-3\gamma^3 + 6\gamma^2(1+\eta) - 3\gamma\eta(4+\eta) + 2\eta(3\eta + \lambda^2)}{2\gamma\lambda^2}}, \quad (49)$$

$$r_z \equiv \sqrt{\frac{(\gamma - \eta)\lambda^2 - 3(\gamma - \eta)^2}{\gamma\lambda^2}}. \quad (50)$$

To study the stability of these critical points, we obtain $\delta C_1 = -\frac{3}{2}\eta\bar{x}^{-2}\bar{z}^2\delta x + 3\eta\bar{x}^{-1}\bar{z}\delta z$ and $\delta C_2 = \frac{3}{2}\eta\delta z$ by linearizing C_1 and C_2 . Substituting δC_1 , δC_2 and the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (37)–(40), we find that Point (E.IV.1) exists and is stable under condition $\eta < -2 + \gamma$ and $\bar{x} > \max\left\{1, \frac{\sqrt{6}}{\lambda}\right\}$; Point (E.IV.2p) exists and is stable under condition $0 > \eta > \gamma - 2$ and $\lambda > \sqrt{\frac{3}{2}(\gamma - \eta)}\sqrt{\frac{\gamma - 2}{\eta}}$; Point (E.IV.2m) is always unstable; Point (E.IV.3p) exists and is stable under condition $\eta < \gamma - 2$ and $\lambda > \sqrt{6}$; Point (E.IV.3m) is always unstable; Point (E.IV.4) exists and is stable under condition $\eta < \gamma - 2$ and $\lambda < \sqrt{6}$; Point (E.IV.5) exists and is stable under condition $\eta < \gamma$ and $\lambda < \min\left\{\sqrt{6}, \sqrt{3(\gamma - \eta)}\right\}$; and Point (E.IV.6) exists and is stable in proper parameter-space [38].

The corresponding Ω_h , Ω_m , w_h and w_{eff} of the attractors are presented in Table IV as well. Once again, we see that their EoS of hessence and effective EoS are all larger than -1 [note that $\eta < 0$ is required by the existence of its corresponding \bar{x} for Point (E.IV.2p), while $\gamma - \eta > 0$ is required by Eq. (50) for Point (E.IV.6)].

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$	Ω_h	Ω_m	w_h	w_{eff}
E.IV.1	$\bar{x}^2 \geq 1, 0, 0, 0, \pm\sqrt{\bar{x}^2 - 1}$	1	0	1	1
E.IV.2p	$\sqrt{\frac{\eta}{\gamma-2}}, 0, \sqrt{1 - \frac{\eta}{\gamma-2}}, 0, 0$	$\frac{\eta}{\gamma-2}$	$1 - \frac{\eta}{\gamma-2}$	1	$-1 + \gamma - \eta$
E.IV.2m	$-\sqrt{\frac{\eta}{\gamma-2}}, 0, \sqrt{1 - \frac{\eta}{\gamma-2}}, 0, 0$	—	—	—	—
E.IV.3p	$+1, 0, 0, 0, 0$	1	0	1	1
E.IV.3m	$-1, 0, 0, 0, 0$	—	—	—	—
E.IV.4	$\frac{\sqrt{6}}{\lambda}, 0, 0, 0, \pm\sqrt{\frac{6}{\lambda^2} - 1}$	1	0	1	1
E.IV.5	$\frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}}, 0, 0, 0$	1	0	$-1 + \frac{\lambda^2}{3}$	$-1 + \frac{\lambda^2}{3}$
E.IV.6	$\sqrt{\frac{3}{2}\frac{\gamma-\eta}{\lambda}}, r_y, r_z, 0, 0$	$1 - r_z^2$	r_z^2	$-1 + \frac{3(\gamma-\eta)^2}{\lambda^2(1-r_z^2)}$	$-1 + \gamma - \eta$

TABLE IV: Critical points for Case (IV) $C = 3\eta H\rho_m$ in the model with exponential potential. r_y and r_z are given in Eqs. (49) and (50), respectively.

V. MODEL WITH (INVERSE) POWER LAW POTENTIAL

In this section, we consider the hessence model with a (inverse) power law potential

$$V(\phi) = V_0 (\kappa\phi)^n, \quad (51)$$

where n is a dimensionless constant. $V(\phi)$ is a power law potential when $n > 0$ while it is an inverse power law potential when $n < 0$. In this case, Eqs. (21)–(25) become

$$x' = 3x \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - 1 \right) - uv^2 - \frac{n}{2} uy^2 - C_1, \quad (52)$$

$$y' = 3y \left(x^2 - v^2 + \frac{\gamma}{2} z^2 \right) + \frac{n}{2} uxy, \quad (53)$$

$$z' = 3z \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - \frac{\gamma}{2} \right) + C_2, \quad (54)$$

$$u' = -xu^2, \quad (55)$$

$$v' = 3v \left(x^2 - v^2 + \frac{\gamma}{2} z^2 - 1 \right) - xuv, \quad (56)$$

where C_1 and C_2 are defined in Eq. (26) and depend on interaction form C . In the following subsections, we will consider four interaction forms C given in the end of Sec. III.

To study the stability of the critical points of Eqs. (52)–(56), we substitute linear perturbations $x \rightarrow \bar{x} + \delta x$, $y \rightarrow \bar{y} + \delta y$, $z \rightarrow \bar{z} + \delta z$, $u \rightarrow \bar{u} + \delta u$, and $v \rightarrow \bar{v} + \delta v$ about the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (52)–(56) and linearize them. Note that these critical points must satisfy the Friedmann constraint, $\bar{y} \geq 0$, $\bar{z} \geq 0$ and requirement of \bar{x} , \bar{y} , \bar{z} , \bar{u} , \bar{v} all being real. Because of the Friedmann constraint (28), there are only four independent evolution equations:

$$\begin{aligned} \delta x' = & \left\{ 3 \left[\left(\frac{\gamma}{2} - 1 \right) \bar{z}^2 - \bar{y}^2 \right] - 2\bar{u}\bar{x} \right\} \delta x - [6\bar{x}\bar{y} + (2+n)\bar{u}\bar{y}] \delta y + [3(\gamma-2)\bar{x} - 2\bar{u}] \bar{z} \delta z \\ & - \left[\bar{x}^2 + \left(1 + \frac{n}{2} \right) \bar{y}^2 + \bar{z}^2 - 1 \right] \delta u - \delta C_1, \end{aligned} \quad (57)$$

$$\delta y' = \frac{n}{2} \bar{y} \bar{u} \delta x + 3 \left[1 - 3\bar{y}^2 + \left(\frac{\gamma}{2} - 1 \right) \bar{z}^2 + \frac{n}{6} \bar{u} \bar{x} \right] \delta y + 3(\gamma-2) \bar{y} \bar{z} \delta z + \frac{n}{2} \bar{x} \bar{y} \delta u, \quad (58)$$

$$\delta z' = -6\bar{y} \bar{z} \delta y + \left\{ (\gamma-2) \bar{z} + 3 \left[\left(1 - \frac{\gamma}{2} \right) - \bar{y}^2 + \left(\frac{\gamma}{2} - 1 \right) \bar{z}^2 \right] \right\} \delta z + \delta C_2, \quad (59)$$

$$\delta u' = -\bar{u}^2 \delta x - 2\bar{x} \bar{u} \delta u, \quad (60)$$

where δC_1 and δC_2 are the linear perturbations coming from C_1 and C_2 , respectively. Again, the four eigenvalues of the coefficient matrix of the above equations determine the stability of the critical points.

A. Case (I) $C = 0$

In this case, $C_1 = C_2 = 0$, namely there is *no* interaction between hessence and background matter. It is easy to find out all critical points $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ of the autonomous system (52)–(56). We present them in Table V. Checking the eigenvalues of Eqs.(57)–(60), we find that Points (P.I.1), (P.I.2), and (P.I.4) are always unstable, while Point (P.I.3) is always stable. The unique late time attractor (P.I.3) has

$$\Omega_h = 1, \quad \Omega_m = 0, \quad w_h = -1, \quad w_{eff} = -1, \quad (61)$$

which is a hessence-dominated solution and it is an asymptotical de Sitter attractor.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$
P.I.1	$\bar{x}^2 \geq 1, 0, 0, 0, \pm\sqrt{\bar{x}^2 - 1}$
P.I.2	$0, 0, 1, \text{any}, 0$
P.I.3	$0, 1, 0, 0, 0$
P.I.4	$\pm 1, 0, 0, 0, 0$

TABLE V: Critical points for Case (I) $C = 0$ in the model with (inverse) power law potential.

B. Case (II) $C = \alpha\kappa\rho_m\dot{\phi}$

In this case, $C_1 = \sqrt{\frac{3}{2}}\alpha z^2$ and $C_2 = \sqrt{\frac{3}{2}}\alpha xz$. We summarize the physically reasonable critical points of the autonomous system (52)–(56) in Table VI. Next, we consider the stabilities of these critical points. In this case, $\delta C_1 = \sqrt{6}\alpha\bar{z}\delta z$ and $\delta C_2 = \sqrt{\frac{3}{2}}\alpha\bar{z}\delta x + \sqrt{\frac{3}{2}}\alpha\bar{x}\delta z$. The four eigenvalues of the coefficient matrix of Eqs. (57)–(60) tell us that Points (P.II.1), (P.II.2), (P.II.3) and (P.II.4) are always unstable, while Point (P.II.5) is always stable. The unique late time attractor (P.II.5) has

$$\Omega_h = 1, \quad \Omega_m = 0, \quad w_h = -1, \quad w_{eff} = -1, \quad (62)$$

which is a hessence-dominated solution and it is an asymptotical de Sitter attractor.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$
P.II.1	$\bar{x}^2 \geq 1, 0, 0, 0, \pm\sqrt{\bar{x}^2 - 1}$
P.II.2	$\pm 1, 0, 0, 0, 0$
P.II.3	$\sqrt{\frac{2}{3}\frac{\alpha}{\gamma-2}}, 0, \sqrt{1 - \frac{2\alpha^2}{3(\gamma-2)^2}}, 0, 0$
P.II.4	$\sqrt{\frac{3}{2}\frac{\gamma-2}{\alpha}}, 0, 0, 0, \pm\sqrt{-1 + \frac{3(\gamma-2)^2}{2\alpha^2}}$
P.II.5	$0, 1, 0, 0, 0$

TABLE VI: Critical points for Case (II) $C = \alpha\kappa\rho_m\dot{\phi}$ in the model with (inverse) power law potential.

C. Case (III) $C = 3\beta H\rho_{tot} = 3\beta H(\rho_h + \rho_m)$

In this case, $C_1 = \frac{3}{2}\beta x^{-1}$ and $C_2 = \frac{3}{2}\beta z^{-1}$. The physically reasonable critical points of the autonomous system (52)–(56) are summarized in Table VII. There, r_3 is defined by Eq. (43). Substituting $\delta C_1 = -\frac{3}{2}\beta\bar{x}^{-2}\delta x$, $\delta C_2 = -\frac{3}{2}\beta\bar{z}^{-2}\delta z$, and the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (57)–(60), we find that *no* stable attractor exists in this case.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$
P.III.1p	$[\frac{1}{2}(1+r_3)]^{1/2}, 0, [\frac{1}{2}(1-r_3)]^{1/2}, 0, 0$
P.III.1m	$-\frac{1}{2}(1+r_3)]^{1/2}, 0, [\frac{1}{2}(1-r_3)]^{1/2}, 0, 0$
P.III.2p	$[\frac{1}{2}(1-r_3)]^{1/2}, 0, [\frac{1}{2}(1+r_3)]^{1/2}, 0, 0$
P.III.2m	$-\frac{1}{2}(1-r_3)]^{1/2}, 0, [\frac{1}{2}(1+r_3)]^{1/2}, 0, 0$

TABLE VII: Critical points for Case (III) $C = 3\beta H\rho_{tot} = 3\beta H(\rho_h + \rho_m)$ in the model with (inverse) power law potential. r_3 is given in Eq. (43).

D. Case (IV) $C = 3\eta H\rho_m$

In this case, $C_1 = \frac{3}{2}\eta x^{-1}z^2$ and $C_2 = \frac{3}{2}\eta z$, we have the physically reasonable critical points shown in Table VIII. Substituting $\delta C_1 = -\frac{3}{2}\eta\bar{x}^{-2}\bar{z}^2\delta x + 3\eta\bar{x}^{-1}\bar{z}\delta z$, $\delta C_2 = \frac{3}{2}\eta\delta z$, and the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into Eqs. (57)–(60), the four eigenvalues of the coefficient matrix of the resulting equations tell us that *no* stable attractor exists as well in this case.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$
P.IV.1	$\bar{x}^2 \geq 1, 0, 0, 0, \pm\sqrt{\bar{x}^2 - 1}$
P.IV.2	$\pm\sqrt{\frac{\eta}{\gamma-2}}, 0, \sqrt{1 - \frac{\eta}{\gamma-2}}, 0, 0$
P.IV.3	$\pm 1, 0, 0, 0, 0$

TABLE VIII: Critical points for Case (IV) $C = 3\eta H\rho_m$ in the model with (inverse) power law potential.

VI. NO BIG RIP IN HESSENCE MODEL

Obviously, we can see from Sec. IV and Sec. V that w_h and w_{eff} of all stable late time attractors are larger than or equal to -1 for these two particular models with exponential or (inverse) power law potential and the interaction term C is chosen to be four different forms. No phantom-like late time attractor with w_h or w_{eff} less than -1 can exist in the hessence model of dark energy. However, one may wonder whether this observation depends on the forms of potential $V(\phi)$ and the interaction term C . Therefore, it is interesting to investigate this feature in a general case.

Let us come back to the most general equations system, i.e. Eqs. (21)–(25). Now, we leave $V(\phi)$ and C undetermined, except for assuming that they can make the equations system closed, in other words, Eqs. (21)–(25) is an autonomous system. The critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ of the autonomous system satisfies $\bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = \bar{v}' = 0$. Of course, they are also subject to the Friedmann constraint, i.e. $\bar{x}^2 + \bar{y}^2 + \bar{z}^2 - \bar{v}^2 = 1$. From Eq. (24), $\bar{u}' = 0$, we have $\bar{x}\bar{u} = 0$. Substituting this into Eq. (25), $\bar{v}' = 0$, we have either $\bar{v} = 0$ or $\bar{x}^2 - \bar{v}^2 + \frac{\gamma}{2}\bar{z}^2 - 1 = 0$. If the latter holds, from Eq. (29), we have $\Omega_m = \bar{z}^2 = \frac{2}{\gamma} [1 - (\bar{x}^2 - \bar{v}^2)]$. Physics requires $\Omega_m \leq 1$. Therefore $\bar{x}^2 - \bar{v}^2 \geq 0$ is required for the case $0 < \gamma \leq 2$. We see from Eq. (30) that $w_h \geq -1$ in this case. If $\bar{x}^2 - \bar{v}^2 + \frac{\gamma}{2}\bar{z}^2 - 1 \neq 0$, we then have $\bar{v} = 0$ as mentioned above. Obviously, from Eq. (30), $w_h \geq -1$ is inevitable. On the other hand, we can see no big rip in the hessence model as follows. Since $\bar{x}^2 - \bar{v}^2 \geq 0$ for both cases mentioned above, we can see from Eq. (27) that $\dot{H} \leq 0$, which implies $w_{eff} \geq -1$, cf. Eq. (16). Therefore, we conclude that $w_h \geq -1$ and $w_{eff} \geq -1$ always hold for all critical points of the autonomous system (21)–(25). No phantom-like late time attractor with EoS less than -1 can exist. This result is independent of the form of potential $V(\phi)$ and interaction form C .

Therefore, EoS less than -1 is transient in the hessence model. Eventually, it will go to quintessence-like attractors whose EoS is larger than -1 or asymptotically to de Sitter attractor whose EoS is a constant -1 . Thus, the big rip will not appear in the hessence model.

VII. CONCLUSION AND DISCUSSIONS

In this work, the cosmological evolution of hessence dark energy is investigated. We considered two models with exponential and (inverse) power law potentials of hessence respectively, and investigated the dynamical system for the four cases with different interactions between hessence and background perfect fluid. By the phase-space analysis, we find that some stable attractors can exist, which are either scaling solutions or hessence-dominated solutions with EoS larger than or equal to -1 . No phantom-like late time attractors with EoS less than -1 can exist. We have shown that this essential result still holds in a general case beyond particular potentials and interaction forms. Thus, the big rip will not appear in the hessence model.

Our result of the hessence model is very different from that of the quintom model studied in [22, 23], where the phantom-dominated solution is the unique late time attractor and the big rip is inevitable. The difference between our result and that of [22, 23] is not due to the interaction between the hessence and background matter, since our conclusion still holds for the case $C = 0$, that is, the case without the interaction. The difference should be due to the form of the interaction between ϕ_1 and ϕ_2 [cf. Eqs. (4)–(7)]. Guo *et al* [22] and Zhang *et al* [23] only studied the cosmological evolution of the quintom model without direct coupling between ϕ_1 and ϕ_2 , i.e. $V(\phi_1, \phi_2) = V_{\phi_1} + V_{\phi_2} \equiv V_{\phi_1 0} \exp(-\lambda_{\phi_1} \kappa \phi_1) + V_{\phi_2 0} \exp(-\lambda_{\phi_2} \kappa \phi_2)$, and with a special interaction between ϕ_1 and ϕ_2 , i.e. $V(\phi_1, \phi_2) = V_{\phi_1} + V_{\phi_2} + V_{int}$ and $V_{int} \sim (V_{\phi_1} V_{\phi_2})^{1/2}$, respectively. As pointed out in [28], in the hessence model, the potential is

imposed to be the form of $V(\phi)$, or equivalently, $V(\phi_1^2 - \phi_2^2)$ in terms of ϕ_1 and ϕ_2 [cf. Eqs. (4)–(7)]. Except for the very special case with $V(\phi) \sim \phi^2$, the two fields ϕ_1 and ϕ_2 are coupled in the general case, and the interaction between ϕ_1 and ϕ_2 is quite complicated, rather than the very special interaction considered in [23].

Another issue is about the fate of our universe. From our result, the present super-acceleration of universe, i.e. $\dot{H} > 0$, is transient and our universe can avoid the fate of big rip. It can either accelerate forever ($w_{eff} < -1/3$) or come back to deceleration ($w_{eff} > -1/3$), which is determined by the model parameters.

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